Prove $\dfrac{\cot C - \tan C}{\cos C + \sin C} = \dfrac{\cos C - \sin C}{\cos C \sin C}$.

$$= \frac{\frac{\cos C}{\sin C} - \frac{\sin C}{\cos C}}{\cos C + \sin C} \cdot \frac{\sin C \cos C}{\sin C \cos C}$$

$$= \frac{\cos^2 C - \sin^2 C}{(\cos C + \sin C)\sin C \cos C}$$

$$= \frac{(\cos C - \sin C)\cancel{(\cos C + \sin C)}}{\cancel{(\cos C + \sin C)}\sin C \cos C}$$

$$= \frac{\cos C - \sin C}{\sin C \cos C}$$

$$\frac{\cos\theta}{\tan\theta+\sec\theta}$$

$$= \frac{\cos\theta}{\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}} \cdot \frac{\frac{\cos\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}}$$

$$= \frac{\cos^2\theta}{\sin\theta+1}$$

$$= \frac{1-\sin^2\theta}{\sin\theta+1}$$

$$= \frac{(1-\sin\theta)(1+\sin\theta)}{\sin\theta+1}$$

$$= 1-\sin\theta$$

$$\frac{1}{(1-\sec^2\beta)\cos\beta}$$

$$= \frac{1}{-\tan^2\beta\cos\beta}$$

$$= \frac{1}{-\frac{\sin^2\beta}{\cos^2\beta}\cos\beta}$$

$$= -\frac{\cos\beta}{\sin^2\beta}$$

$$= -\frac{\cos\beta}{\sin\beta}\frac{1}{\sin\beta}$$

$$= -\cot\beta\csc\beta$$

$$\frac{\cos x}{1-\sin x} - \frac{1-\sin x}{\cos x} = 2\tan x$$

$$= \frac{\cos^2 x - (1-\sin x)^2}{(1-\sin x)\cos x}$$

$$= \frac{\cos^2 x - (1 - 2\sin x + \sin^2 x)}{(1-\sin x)\cos x}$$

$$= \frac{\cancel{1} - \sin^2 x - \cancel{1} + 2\sin x - \sin^2 x}{(1-\sin x)\cos x}$$

$$= \frac{2\sin x - 2\sin^2 x}{(1-\sin x)\cos x}$$

$$= \frac{2\sin x(1-\sin x)}{(1-\sin x)\cos x}$$

$$= 2\tan x$$

$\csc^2 A + \sec^2 A = (\cot A + \tan A)^2$

$\downarrow$

$= \cot^2 A + 2\cot A \tan A + \tan^2 A$

$= \csc^2 A - 1 + 2\frac{1}{\tan A}\tan A + \sec^2 A - 1$

$= \csc^2 A + \sec^2 A$